HW. # 13

Homework problems are taken from several textbooks. The problems are color coded to indicate level of difficulty. The color green indicates an elementary problem, which you should be able to solve effortlessly. Yellow means that the problem is somewhat harder. Red indicates that the problem is hard. You should attempt the hard problems especially.

Find all of the critical points of the given function.

1. $f(x,y) = 2x^2 + xy - 4y^2$
2. $f(x,y) = xy(x + y - 1)$
3. $f(x,y) = (5x - 2y)^2$
4. $f(x,y) = (x^2 - 1)(y^2 - 1)$
5. $f(x,y) = xy^{\frac{2}{3}}$
6. $f(x,y) = Cos(x)Sin(y)$
7. $f(x,y) = e^{-x^2 - y^2}$

Use the second derivative test to identify the local extrema of the functions below.

8. $f(x,y) = x^2 + y^2 + 2x - y + 3$
9. $f(x,y) = -3x^2 + 6xy + 2y^2 + 12x - 12y$
10. $f(x,y) = x^3 + xy^2 - y^2 - x$.
11. $f(x,y) = (x^2 - x)(y^2 - y)$
12. $f(x,y) = x^4 - 3x^2y^2 + y^4$
13. $f(x,y) = (x^2 + y^2)e^{-y}$

14. $f(x, y) = x + y - \ln(xy)$

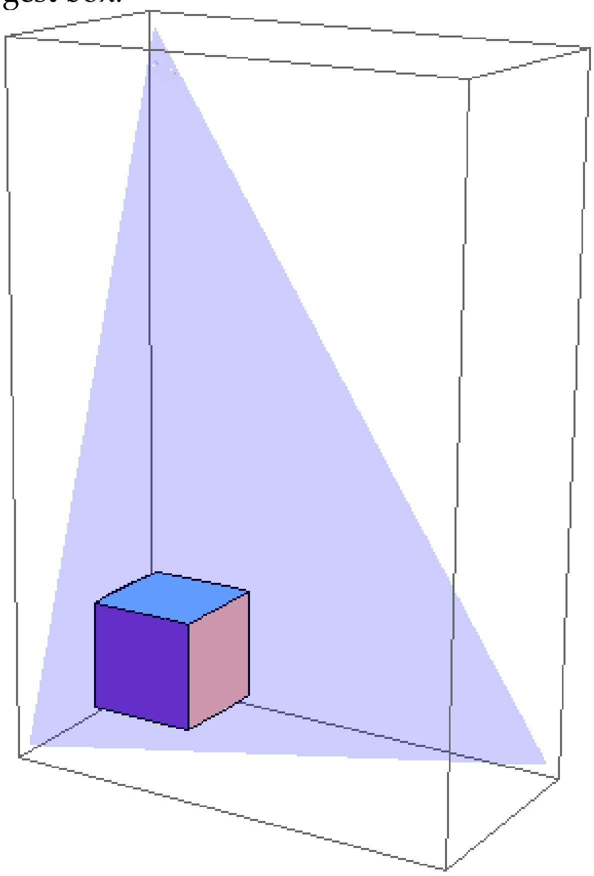
15. $f(x, y) = x^2 + y^2 + z^2 + yz$

16. Find a point on the plane z = 4x + 3y + 1 that is closest to the origin.

17. Find the points on the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$ that are closest to the origin.

18. Find the pair of points on the skew lines f(s) = (2s, 3s – 1, -s + 2) and g(t) = (t + 1, -t +3, 4t + 5) that are closest to one another.

19. A rectangular box is to be inscribed in the tetrahedron whose faces are the coordinate planes and the plane x/a + y/b + z/c = 1 (Here a, b, and c are given positive constants.) One corner of the box touches the plane, the opposite corner is at the origin, and the faces of the box are parallel to the coordinate planes. Find the dimensions of the largest box.



20. A closed rectangular box is to contain 1000 cm^3. If the material for the top and bottom costs 2 cents per square centimeter and the material for the sides costs 4 cents per square centimeter, find the dimensions of the box that minimize its cost.

21. For a set of data (x_1, y_1), (x_2, y_2),..., (x_n, y_n), the **least squares regression line** is the line y = m x + b, where m and b are chosen to minimize the sum

$$S(m,b) = \sum_{i=1}^{n} (y_i - mx_i - b)^2 .$$

Show that the values of m and b that minimize this sum are

$$m = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{\sum_{i=1}^{n} x_i^2 - n\left(\overline{x}\right)^2}$$

and

$$b = \overline{y} - m\overline{x}$$

where $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ are the averages of the x's and y's.